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## Solution

Q1. $x = a\cos^3 t \qquad y = a\sin^3 t$

$$\frac{dx}{dt} = 3a\cos^2 t \sin t \qquad \frac{dy}{dt} = 3a\sin^2 t \cos t$$

$$\frac{dy}{dx} = \frac{3a\sin^2 t \cos t}{3a\cos^2 t \sin t} = \tan t$$

at $t = \frac{3\pi}{4}$

$$\frac{dy}{dx} = \tan\frac{3\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = -\tan\frac{\pi}{4}$$

$$= -1$$

Option (a) is correct

Q2. $$y = \frac{1}{1 + x^{a-b} + x^{c-b}} + \frac{1}{1 + x^{b-c} + x^{a-c}} + \frac{1}{1 + x^{b-a} + x^{c-a}}$$

$$y = \frac{1}{1 + \frac{x^a}{x^b} + \frac{x^c}{x^b}} + \frac{1}{1 + \frac{x^b}{x^c} + \frac{x^a}{x^c}} + \frac{1}{1 + \frac{x^b}{x^a} + \frac{x^c}{x^a}}$$

$$y = \frac{x^b}{x^b + x^a + x^c} + \frac{x^c}{x^c + x^b + x^a} + \frac{x^a}{x^a + x^b + x^c}$$

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$$y = \frac{x^a + x^b + x^c}{x^a + x^b + x^c}$$

$y = 1$

$\frac{dy}{dx} = 0$ option (b) is correct

Q3. $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$x\sqrt{1+y} = -y\sqrt{1+x}$

S.B.S

$x^2(1+y) = y^2(1+x)$

$x^2 + x^2y = y^2 + y^2x$

$(x^2 - y^2) + (x^2y - y^2x) = 0$

$(x+y)(x-y) + xy(x-y) = 0$

$(x-y)(x+y+xy) = 0$

Either $x - y = 0$ O.R. $x + y + xy = 0$

$x = y$

Rejected ∵ given $x \neq y$

$x + y(1+x) = 0$

$y(1+x) = -x$

$y = \frac{-x}{1+x}$

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$$y = \frac{-x}{1+x}$$

D.B.S

$$\frac{dy}{dx} = -\left[\frac{(1+x) - x}{(1+x)^2}\right]$$

$$\frac{dy}{dx} = \frac{-1}{(1+x)^2}$$ proved.

Q4. $$y = x \log\left(\frac{x}{a+bx}\right)$$

$$\frac{y}{x} = \log\left(\frac{x}{a+bx}\right)$$

$$\frac{y}{x} = \log x - \log(a+bx)$$

D.B.S

$$\frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{b}{a+bx}$$

$$\frac{x\frac{dy}{dx} - y}{x^2} = \frac{a+bx-bx}{x(a+bx)}$$

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$$x\frac{dy}{dx} - y = \frac{ax}{a+bx} \quad \text{---(1)}$$

Again D.B.w.r.t x

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(a+bx)a - axb}{(a+bx)^2}$$

$$x\frac{d^2y}{dx^2} = \frac{a^2 + abx - abx}{(a+bx)^2}$$

Mult. both sides by $x^2$

$$x^3\frac{d^2y}{dx^2} = \frac{a^2x^2}{(a+bx)^2}$$

$$\Rightarrow y_2 = \left(\frac{ax}{a+bx}\right)^2$$

$$\Rightarrow y_2 = \left(x\frac{dy}{dx} - y\right)^2 \text{ (using eq (1))}$$

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Q5 $x = \sqrt{a^{\sin^{-1}t}}$ $\quad y = \sqrt{a^{\cos^{-1}t}}$

S.B.S

$x^2 = a^{\sin^{-1}t}$ — (1)

$2x\frac{dx}{dt} = a^{\sin^{-1}t} \log a \cdot \frac{1}{\sqrt{1-t^2}}$

$2x\frac{dx}{dt} = \frac{x \log a}{\sqrt{1-t^2}}$ (using eq. (1))

$$\boxed{\frac{dx}{dt} = \frac{x \log a}{2\sqrt{1-t^2}}}$$

Now $y = \sqrt{a^{\cos^{-1}t}}$

S.B.S

$y^2 = a^{\cos^{-1}t}$ — (2)

$2y\frac{dy}{dt} = a^{\cos^{-1}t} \log a \left(\frac{-1}{\sqrt{1-t^2}}\right)$

$2y\frac{dy}{dt} = \frac{-y^2 \log a}{\sqrt{1-t^2}}$ (using eq. (2))

$$\boxed{\frac{dy}{dt} = \frac{-y \log a}{2\sqrt{1-t^2}}}$$

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$$\frac{dy}{dx} = \frac{-y \log a}{2\sqrt{1-t^2}} \cdot \frac{x \log a}{2\sqrt{1-t^2}}$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

Q6 $(x-a)^2 + (y-b)^2 = c^2$ — (1)

D.B.S

$$2(x-a) + 2(y-b)\frac{dy}{dx} = 0$$

$$(x-a) + (y-b)\frac{dy}{dx} = 0$$

$$(y-b)\frac{dy}{dx} = -(x-a)$$

$$\frac{dy}{dx} = -\frac{(x-a)}{y-b} \quad \text{— (2)}$$

Again D.B.S

$$\frac{d^2y}{dx^2} = -\left[\frac{(y-b) - (x-a)\frac{dy}{dx}}{(y-b)^2}\right.$$

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$$\frac{d^2y}{dx^2} = -\frac{\left[(y-b) + \frac{(x-a)^2}{(y-b)}\right]}{(y-b)^2} \quad \text{(using eq. 2)}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{(y-b)^2 + (x-a)^2}{(y-b)^3}\right]$$

$$\frac{d^2y}{dx^2} = -\frac{c^2}{(y-b)^3} \quad (3) \quad \text{(using eq (1))}$$

Now

$$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + \frac{(x-a)^2}{(y-b)^2}\right]^{\frac{3}{2}}}{\frac{-c^2}{(y-b)^3}} \quad \text{(using 2 \& 3)}$$

$$= \frac{\left[\frac{(y-b)^2 + (x-a)^2}{(y-b)^2}\right]^{\frac{3}{2}}}{\frac{-c^2}{(y-b)^3}}$$

$$= \left[\frac{c^2}{(y-b)^2}\right]^{\frac{3}{2}} \Big/ \frac{-c^2}{(y-b)^3} = \frac{c^3}{(y-b)^3} \Big/ \frac{-c^2}{(y-b)^3} = -c$$

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(7) $e^y = y^x$

Taking log

$\log e^y = \log y^x$

$y \log e = x \log y$

$y = x \log y \quad (\because \log e = 1)$

$x = \frac{y}{\log y}$

$\frac{dx}{dy} = \frac{\log y - y \cdot \frac{1}{y}}{(\log y)^2}$

$\frac{dx}{dy} = \frac{\log y - 1}{(\log y)^2}$

$\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$

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